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MITIGATION OF THREE-DIMENSIONAL VIBRATION OF INCLINED SAG CABLE USING DISCRETE OIL DAMPERS—II. APPLICATION

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The hybrid method developed in part I of this paper for three dimensional free and forced vibrations of an inclined sag cable with multi-pairs of oil dampers is now applied to cables in a real cable stayed bridge. Damper performance is evaluated in terms of the achievable maximum modal damping ratio and the possible maximum reduction of resonant response of a cable in both in-plane and out-of-plane vibrations. Effects of both cable properties and damper parameters, such as cable sag, cable inclination, damper stiffness, and damper location, are discussed. Optimum parameters for oil dampers, such as damper damping coefficient and damper direction, are sought within a range of practical interest. The sensitivity study is also carried out to see the influence of small deviation of damper direction on damper performance. Finally, an investigation is performed to see how a proper arrangement of two pairs of oil dampers can overcome insufficient damping ratio problem in the first in-plane mode of a long cable due to frequency avoidance.

1. INTRODUCTION

Most previous studies on cable vibration mitigation using oil damper in cable stayed bridges are based on the model of a horizontal taut string with the oil damper normal to the string near its support [1–5]. This kind of model ignores some inherent properties of both cable and damper, such as cable sag, cable inclination, and damper stiffness. It also considers only in-plane vibration of the cable-damper system. Cables in a long span cable stayed bridge are actually inclined sag cables. When such a cable vibrates in its static equilibrium plane due to vortex shedding, wind–rain excitation or others, the cable also experiences buffeting due to turbulence in incident wind. Thus, the vibration of cable in a long span cable stayed bridge is in fact a three-dimensional inclined sag cable vibration. Furthermore, in practice most oil dampers for reducing cable vibration are installed in pair and herringbonely to the cable near the deck support. Therefore, a three-dimensional dynamic analysis of the damper-inclined sag cable system is desirable in order to capture dynamic characteristics and dynamic behavior of the system and to clarify the differences in results between three-dimensional and two-dimensional analyses.

In part I of this paper, three dimensional free and forced vibration problems of an inclined sag cable with multi-pairs of oil dampers have been formulated using a hybrid method. A computer program has been written according to the derived formulae and has been verified against Irvine's theory in the case of a single horizontal cable [6]. The method and the program are now applied to inclined cables in a long span cable stayed bridge, which is located in a typhoon- and rain-prone region. This bridge employs three single column towers to support its deck through 384 stay cables, forming two main spans of

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448 m and 475 m, respectively, and two side spans of 127 m each. The 384 stay cables are arranged in four cable planes: two planes located near the center of the deck; and the other two planes positioned at the outer edges of the deck. Apart from these stay cables, there are additional 460 m long stay cables from the top of the middle tower to the deck cross beams near the two side towers.

Two typical cables with one from 384 stay cables and the other from additional long stay cables are selected for detailed studies here. Vibrational amplitudes of the cable are assumed to be small because of the installation of oil dampers. For each cable, the performance of a pair of oil dampers herringbonely installed to the cable near the bridge deck is first investigated. The effects of cable properties and damper parameters on damper performance are then examined, followed by the sensitivity study of damper performance to damper direction. Finally, the use of two pairs of oil dampers to overcome the insufficient damping problem in the first in-plane mode of the long cable due to frequency avoidance is explored.

The main properties of the two cables are listed in Table 1. An important cable sag parameter λ^2, introduced by Irvine [6], is listed for each cable. This parameter is defined as the ratio of the elastic-to-catenary stiffness. For an inclined cable with small sag-to-span ratio, it can be expressed approximately as

$$\lambda^2 = \left(\frac{mgL}{H}\cos\theta\right)^2 \frac{LEA}{L_e H} \tag{1}$$

where

$$L_e = L_r\left[1 + \left(\frac{mgL}{H}\cos\theta\right)^2 / 8\right] \tag{2}$$

L_r is the distance between two supports of the cable in the $x1$-direction (see Figure 1); m is the mass of the cable per unit length; g is the acceleration due to gravity; H is the horizontal component of the static tension in the x–y plane; L is the horizontal length between two cable end-supports in the x–y plane; E is the cable modulus of elasticity; A is the cross-sectional area of the cable; and the cable inclination θ is defined as

$$\theta = \arccos\frac{L}{L_r}. \tag{3}$$

It is seen that the short cable can be regarded as a taut cable, but the long cable is a cable with significant sag.

TABLE 1

Properties of two typical cables in a long span cable stayed bridge

Cable	L (m)	R (m)	T (N) $\times 10^7$	E (N/m^2) $\times 10^{11}$	A (m^2)	m (kg/m)	Inclination	$\log \lambda^2$
Short Cable	114·7	87·2	1·32	1·95	0·0314	260·62	37·2°	−0·61
Long Cable	442·6	127·0	1·32	1·95	0·0314	260·62	16·0°	0·55

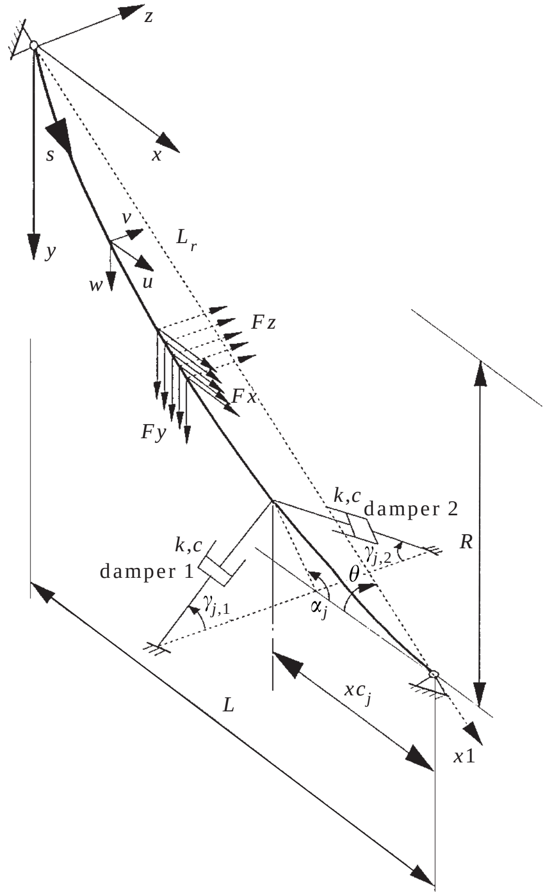

Figure 1. Schematic diagram of an inclined sag cable with oil dampers.

2. DAMPER PERFORMANCE

2.1. SYMMETRIC CASE

Damper performance is evaluated in terms of the achievable maximum modal damping ratio and the possible maximum reduction of resonant response of a cable in both in-plane and out-of-plane vibrations. Symmetric cases are considered first, in which one pair of dampers are installed symmetrically to the cable at a position x_c $(=x_{c1})$ of 2% L with damper direction γ $(=\gamma_{1,1}=\gamma_{1,2})$ of 45° and α $(=\alpha_1)$ of 52·8° for the short cable and 74° for the long cable, respectively (see Figure 1). The purpose of selecting these special angles α is to let the damper forces normal the cable (i.e., normal to line L_r). This position is believed to be an optimum position, which will be discussed later.

Figures 2(a) and (b) show variations of the normalized modal damping ratios in the first five in-plane (the x–y plane) and out-of-plane (the x–z plane) vibrational modes, respectively, of the short cable with the normalized damper sizes. The normalization used here is the same as in the previous studies [4, 7], except that ω_{01} is now selected as the first natural frequency of the out-of-plane cable vibration, which is the lowest natural frequency of the cable. The cable internal damping and damper stiffness are not considered. It is seen from Figure 2(a) that the modal damping ratio curve of the first in-plane mode of vibration is only slightly lower than other four curves, and the other four curves almost overlap each other in the selected non-dimensional coordinates. The modal damping ratio curves of the first five out-of-plane modes of vibration also almost overlap each other, as shown in Figure 2(b). All these curves show that there is an optimum normalized damper size (i.e., damper damping coefficient) about 0·1 for both the in-plane and out-of-plane modes of vibration. The achievable maximum modal damping ratio corresponding to the optimum normalized damper size is about 1% of critical damping. A common optimum damper size for both in-plane and out-of-plane modes of vibration indicates that the symmetric arrangement of a pair of oil dampers is an optimum arrangement in the sense that both in-plane and out-of-plane modes of vibration can have the same maximum modal damping ratio at the same time. Furthermore, both in-plane and out-of-plane modal damping ratio curves of the short cable obtained by the hybrid method were found to be almost the same as the single universal design curve proposed by Pacheco *et al*. for a horizontal taut string with one oil damper [4]. This is because the short cable used here is a taut cable, and the first natural frequency of the in-plane vibration is the same as that of the out-of-plane vibration of the cable.

The performance of a pair of oil dampers can be further examined through forced vibration analysis. Let the short cable carry two uniformly distributed harmonic loads of amplitude 1 N/m: one is normal to the cable line L_r in the cable plane (the x–y plane) and the other is normal to the cable line L_r in the x_1–z plane (see Figure 1). The optimum damper size c of $9{\cdot}21 \times 10^5$ N · s/m found in the free vibration analysis is assigned to each damper to achieve the maximum modal damping ratios in the first in-plane and out-of-plane vibrational modes of the short cable. Figures 3(a) and (b) display the in-plane and out-of-plane response amplitudes, respectively, of the cable at midspan normal to the cable line against excitation frequency for the cases with and without the oil dampers. Within the displayed excitation frequency range, only the first three natural frequencies corresponding to the first three symmetric modes are excited out in either in-plane or out-of-plane vibration because of symmetric harmonic loads considered. Both figures clearly demonstrate that all in-plane and out-of-plane resonance peaks of the cable can be tremendously reduced if the oil dampers of proper parameters are installed to the cable. Once again, the in-plane amplitude response curves are very much similar to the out-of-plane amplitude response curves. These are consistent with the results from the

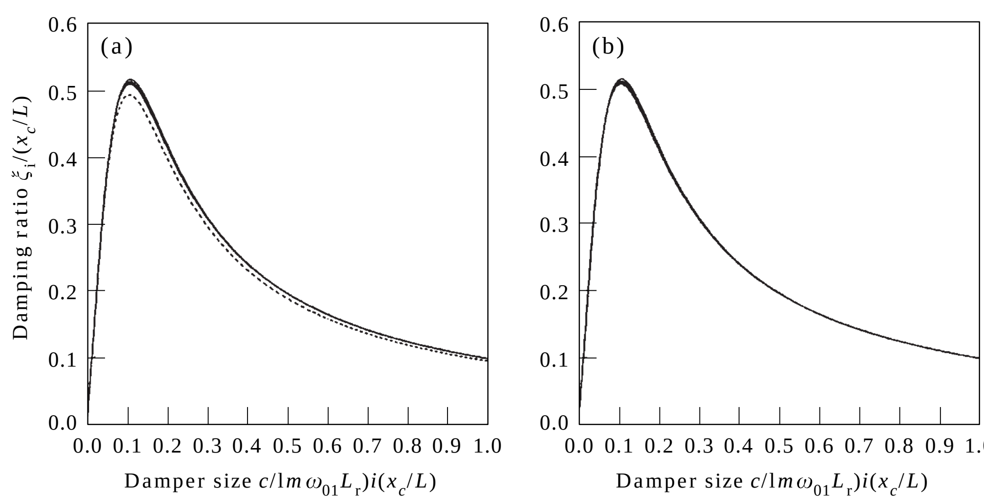

Figure 2. Variations of modal damping ratios of short cable with damper size. (a) In-plane; (b) out-of-plane. – – – Mode 1; —— other modes; short cable, $x_c/L = 0{\cdot}02$, $\gamma = 45°$, $\alpha = 52{\cdot}8°$.

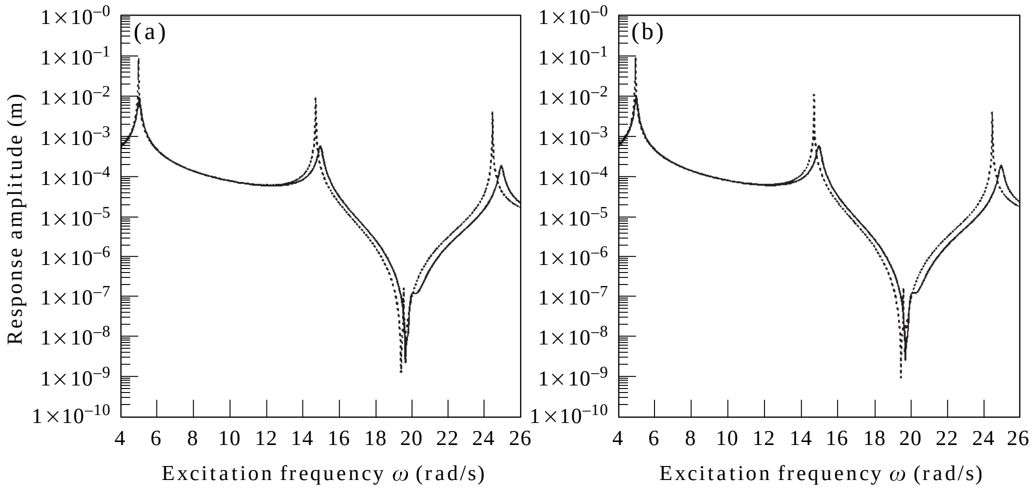

Figure 3. Displacement response at midspan of short cable with and without dampers. (a) In-plane; (b) out-of-plane. – – – Without damper; —— with damper; short cable, $x_c/L = 0{\cdot}02$, $c = 9{\cdot}21 \times 10^5$, $c_1 = c_2 = 2{\cdot}57$.

modal damping ratio analysis. To avoid infinite resonance peaks of the cable in the case without the oil dampers, a small internal damping coefficient $\bar{c}$ $(= c_1 = c_2)$ of $2{\cdot}57$ N · s/m^2 is introduced in the above calculations to both in-plane and out-of-plane forced vibrations of the cable with and without oil dampers.

Displayed in Figures 4(a) and (b) are the normalized modal damping ratios in the first five in-plane and out-of-plane modes of vibration, respectively, of the long cable against the normalized damper size. It is seen that while all other modal damping ratio curves remain the same as the short cable, the modal damping ratio of the first in-plane mode of vibration is significantly reduced. The achievable maximum normalized modal damping ratio in the first in-plane mode is only 0·32 compared with 0·51 for other vibrational modes. The significant reduction of modal damping ratio in the first in-plane vibrational mode

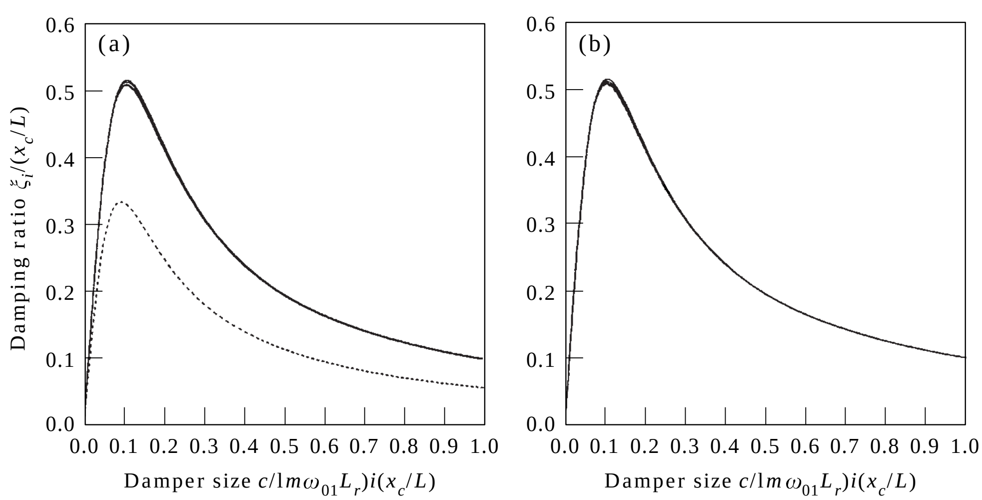

Figure 4. Variations of modal damping ratio of long cable with damper size. (a) In-plane; (b) out-of-plane. – – – Mode 1; —— other modes; long cable, $x_c/L = 0{\cdot}02$; $\gamma = 45°$, $\alpha = 74°$.

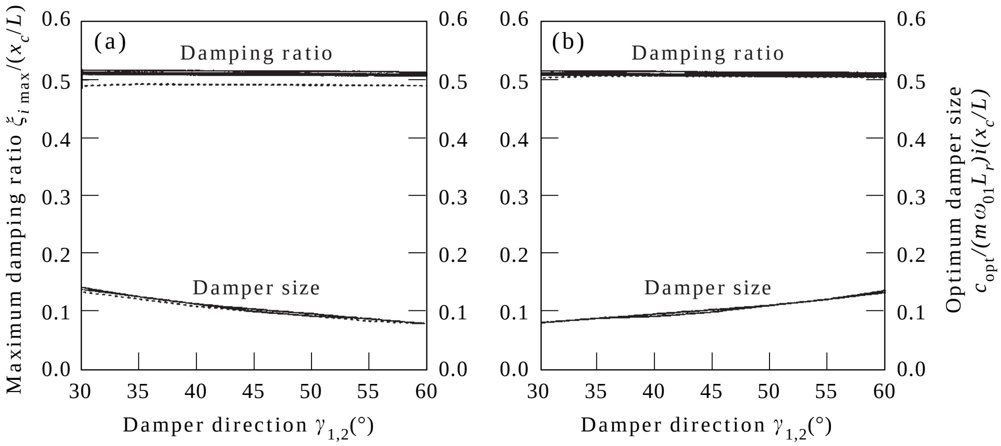

Figure 5. Variations of maximum modal damping ratio and optimum damper size with damper direction γ. (a) In-plane; (b) out-of-plane. – – – Mode 1; —— other modes; short cable, $x_c/L = 0{\cdot}02$.

is due to cable sag effects, which has been discussed by the writers for two-dimensional problems [8] and will be discussed further in Section 3.1.

2.2. UNSYMMETRICAL CASE

In some applications, due to the space limitation around the cable support on the bridge deck (e.g., cables at deck periphery) a pair of oil dampers may have to be installed unsymmetrically with respect to the cable static equilibrium plane. The effects of unsymmetric arrangement of a pair of oil dampers on damper performance are thus investigated here.

Let us consider a case of practical interest in which the direction $\gamma_{1,1}$ of damper 1 is fixed at 45° and the direction $\gamma_{1,2}$ of damper 2 is changed from 30° to 60° via the symmetric position of 45°. All other parameters remain the same as used in Section 2.1. Figures 5(a) and (b) show variations of the achievable maximum modal damping ratio and the corresponding optimum damper size for the first five in-plane and out-of-plane dominated vibrational modes, respectively, of the short cable. It is seen that the maximum normalized modal damping ratio of about 0·5 can still be achieved for both in-plane and out-of-plane dominated modes of vibration within the concerned range of damper direction $\gamma_{1,2}$ if the optimum damper size is adjusted correspondingly. For the in-plane dominated modes of vibration, to maintain the maximum normalized modal damping ratio of about 0·5 as the direction of damper 2 changes from 30° to 60°, the normalized optimum damper size should be adjusted from 0·14 to 0·08. For the out-of-plane dominated modes of vibration, the normalized optimum damper size, however, should be adjusted from 0·08 to 0·14, an opposite way to the in-plane vibration. This indicates that if a pair of oil dampers of the same damping coefficient are unsymmetrically installed to the cable, the maximum modal damping ratio cannot be achieved in both planes at the same time. In most applications, in-plane cable vibration is more severe than out-of-plane cable vibration due to wind or wind–rain excitation. Thus, the optimum damper size may be determined to obtain the maximum in-plane modal damping ratio, i.e., based on Figure 5(a) rather than Figure 5(b). The same observations are also made for the long cable except for the first in-plane vibrational mode, in which the maximum modal damping ratio and the corresponding optimum damper size are significantly smaller than those in other modes due to cable sag effects.

It may be worthwhile to point out that with the unsymmetric installation of a pair of oil dampers, the in-plane and out-of-plane vibrations of the cable are slightly coupled as mentioned in Part I of this paper. As a result, the vibrational mode of the cable is neither a pure in-plane mode nor a pure out-of-plane mode. In fact, it is either an in-plane dominated mode or an out-of-plane dominated mode of vibration.

3. EFFECTS OF CABLE PROPERTIES

Effects of cable sag and cable inclination, which have not been considered in previous studies, on mitigation of three dimensional vibration of inclined sag cables using oil dampers are investigated in this section. Of practical interest, a pair of dampers are considered to be symmetrically installed to the cable at a location x_c of $0{\cdot}02L$ measured from the deck support. The damper direction γ is fixed at 45° and the damper direction α is changed in such a way that as the cable inclination varies, the damper forces are always normal to the cable line L_r.

3.1. EFFECTS OF CABLE SAG

In a horizontal sag cable, if the sag parameter λ^2 is equal to $(2\pi n)^2$ the nth frequency crossover occurs. The frequency of the nth symmetric mode is equal to the frequency of the nth antisymmetric mode; the vertical modal component of the symmetric mode is tangential to the static profile at the supports of the cable; and cross the nth frequency crossover point the order of occurrence of the nth symmetric mode and the nth antisymmetric mode is exchanged. In an inclined sag cable, a frequency avoidance may replace a frequency crossover when the sag parameter λ^2 is equal to $(2\pi n)^2$ [9]. Both the nth symmetric and antisymmetric modes of the cable become hybrid modes, a mixture of symmetric and antisymmetric forms, within a certain region around the frequency avoidance point; and the frequency of the nth symmetric mode is never equal to the nth antisymmetric mode at the frequency avoidance point.

In Section 2.1, it is found that the achievable modal damping ratio in the first in-plane mode of vibration of the long cable is significantly smaller than those in all other in-plane and out-of-plane vibrational modes of the cables. Since this phenomenon does not clearly exist in the short cable, one may assume this is due to frequency avoidance phenomenon associated with cable sag. To confirm this assumption, the maximum modal damping ratios in the first and second in-plane and out-plane vibrational modes of the long cable are calculated for various sag parameters. The change of sag parameter of the long cable is implemented by altering the static tension of the cable while all other cable properties remain unchanged. Both cable inclination θ and damper direction α are set to 45°.

Figure 6(a) shows variations of the maximum modal damping ratios in the first and second in-plane vibrational modes with sag parameter $\log \lambda^2$. The first in-plane mode of vibration of the long cable is a symmetrical mode while the second in-plane mode is an antisymmetrical mode. It is seen that when the inclined cable is taut of the sag parameter less than -1, the maximum normalized modal damping ratio can reach $0{\cdot}5$ in both symmetrical and antisymmetrical modes, which is almost the same as that calculated based on a horizontal taut string with one oil damper [4]. With the increase in sag parameter, the maximum modal damping ratio in the antisymmetrical mode has only a slight change but the maximum modal damping ratio in the symmetric mode is reduced significantly. When the sag parameter equals $1{\cdot}45$, the maximum modal damping ratio in the symmetric mode is reduced to almost zero. This is because the tangential property of the normal component in the first symmetric mode near the deck support entails for the cable a very small motion over there, which has been described in detail by the writers [8]. At the first frequency avoidance of the inclined cable of sag parameter at about 1·6, both the symmetric and antisymmetrical modes become hybrid modes. The maximum normalized modal damping ratio in the first antisymmetrical mode suddenly drops to about 0·25 whereas the maximum normalized modal damping ratio in the first symmetric mode increases from 0 to 0·25. With the further increase of the sag parameter to 1·8, the antisymmetrical mode appears before the symmetric mode. The maximum modal damping ratio in the antisymmetrical mode again is much less affected by frequency avoidance than that in the symmetric mode. The insufficient modal damping ratio in the first in-plane symmetric mode of the long cable due to frequency avoidance may be a proper explanation why the previously-predicted modal damping ratios are larger than the measured results.

Displayed in Figure 6(b) are the maximum modal damping ratios in the first and second out-of-plane vibrational modes of the cable. It is clear that cable sag only slightly affects the maximum modal damping ratios in both modes. When the sag parameter log λ^2 is less than 0·5, the maximum modal damping ratios in both modes retain almost 0·5. As the sag parameter is more than 0·5, the maximum modal damping ratios in both modes start to reduce slightly. This indicates that frequency avoidance of an inclined cable does not significantly affect the modal damping ratios in the out-of-plane modes of vibration.

3.2. EFFECTS OF CABLE INCLINATION

Since the previous studies on oil damper-cable systems of cable stayed bridges are based on a horizontal taut string, effects of cable inclination on the maximum modal damping ratio and the optimum damper size cannot be investigated. The hybrid method described here, however, can be readily used to investigate such effects. Let us fix the length and the static tension of the long cable, assume a pair of oil dampers are always normally and symmetrically installed to the cable at a location x_c/L of 0·02 near the deck support, and then change cable inclination to see how the maximum modal damping ratio and optimum damper size alter. Figure 7(a) shows variations of the maximum modal damping ratio and

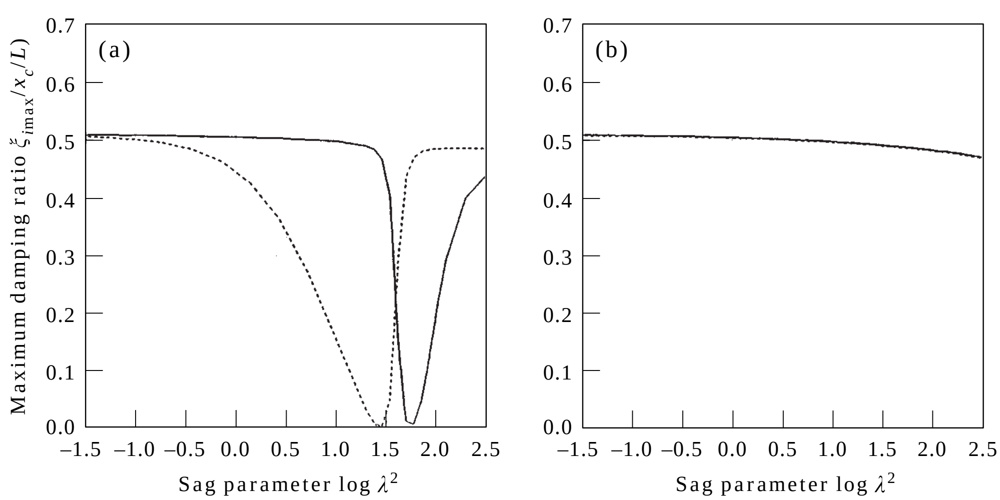

Figure 6. Variations of maximum modal damping ratio with cable sag parameter. (a) In-plane; (b) out-of-plane. – – – Mode 1; —— Mode 2; $\theta = 45°$, $x_c/L = 0{\cdot}02$.

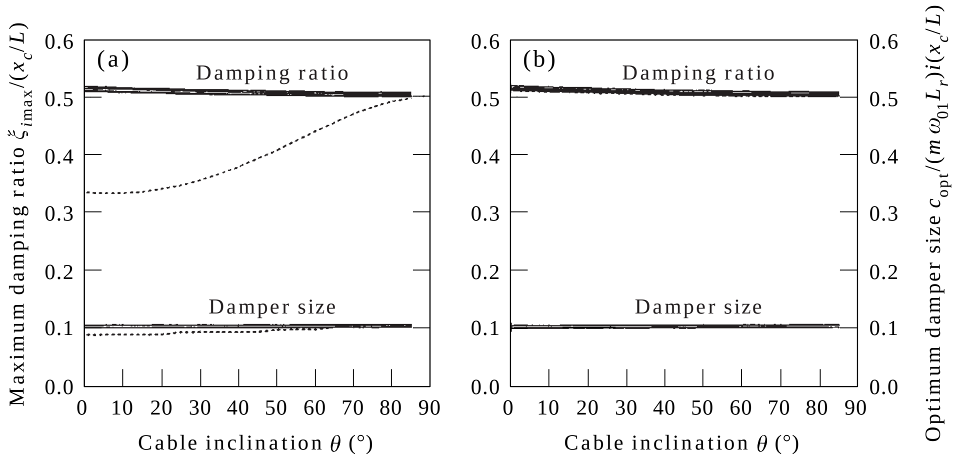

Figure 7. Variations of maximum modal damping ratio and optimum damper size with cable inclination. (a) In-plane; (b) out-of-plane. – – – Mode 1; —— other modes; long cable, $x_c/L = 0{\cdot}02$, $\gamma = 45°$.

the corresponding optimum damper size for the first five in-plane vibrational modes of the long cable with cable inclination while Figure 7(b) displays the same quantities for the first five out-of-plane modes of vibration. It turns out that the maximum modal damping ratios and the optimum damper sizes for both in-plane and out-of-plane modes almost are not affected by the cable inclination except for the first in-plane mode, in which the maximum normalized modal damping ratio is increased from 0·32 for a horizontal cable to 0·50 for an almost vertical cable. The latter results indicate that the effects of frequency crossover or frequency avoidance on the maximum modal damping ratio in the first in-plane symmetric mode of the long cable will be attenuated as cable inclination is increased.

4. EFFECTS OF DAMPER PARAMETERS

Effects of damper location, damper stiffness, and damper direction γ on damper performance are investigated in this section for three dimensional vibration of an inclined sag cable with a pair of oil dampers. The sensitivity of damper performance to the damper direction α for a pair of oil dampers and to the damper direction γ for one in-plane oil damper are also investigated. For a pair of oil dampers, they are assumed to be installed symmetrically and normally to the cable.

4.1. EFFECTS OF DAMPER LOCATION

Due to the physical restriction, the damper cannot be installed far away from the cable support on the deck. Figure 8(a) exhibits the achievable maximum modal damping ratios in the first five in-plane modes of vibration of the short cable against the normalized damper location within a range from 0 to 0·09. Here, the damper stiffness is assumed to be zero and no cable internal damping is considered. It is seen that for any one of the five modes, the maximum modal damping ratio always increases as the distance of the dampers from the deck support becomes larger. When x_c/L is less than 0·02, the achievable maximum modal damping ratios in the first five in-plane modes are almost the same. The ratio of the maximum modal damping ratio to the normalized damper location, i.e., the normalized maximum modal damping ratio used before, is also the same at about 0·5 for each mode as expected. Nevertheless, if the dampers are positioned further away from the deck support, the maximum modal damping ratios in the first five modes become different: the higher the number of mode, the larger the maximum modal damping ratio. The relationship between the achievable maximum modal damping ratio and the normalized damper position becomes non-linear. The reason why the maximum modal damping ratio in the higher mode increases faster than that in the lower one as x_c/L increases is believed to be that near the cable support, the modal displacement and velocity in the higher mode are larger than those in the lower mode. However, if the damper position exceeds a quarter of wave in the higher modes of vibration the above observation, as shown in Figure 8, may not exist since the damper may be located at the node point of vibration mode. It should also be pointed out that as the damper position alters, the damper size should be changed accordingly to achieve the maximum modal damping ratio. Figure 9(a) shows such variations of the optimum damper size with normalized damper position for the first five in-plane modes of the short cable. Clearly, with the increase of the damper position away from the cable support, the optimum damper size decreases. For a given damper position, the optimum damper size required to obtain the maximum modal damping ratio is smaller for the higher mode than for the lower mode. Thus, the determination of damper size mainly depends on the modes of vibration mostly required to be suppressed.

For the same short cable, as expected, variations of the maximum modal damping ratio and the corresponding optimum damper size with the normalized damper location in the out-of-plane modes of vibration are almost the same as those in the in-plane modes of vibration. For the long cable, due to the effect of frequency avoidance the maximum modal damping ratio in the first in-plane vibrational mode is increased much more slowly than those in higher modes as the damper is moved away from the cable support [see Figure 8(b)]. This indicates that even though the damper is moved relatively far away from the cable support, the frequency avoidance still affects the modal damping ratio in the first in-plane vibrational mode of the long cable. Also as expected, the optimum damper sizes for the long cable have only slight difference from those of the short cable except for the first in-plane vibrational modes in which the optimum damper size for the long cable is

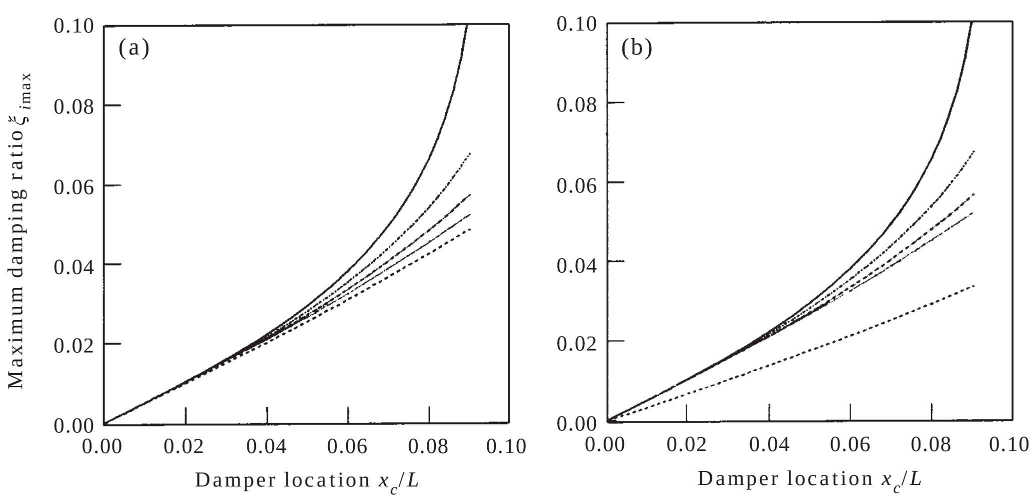

Figure 8. Variations of maximum modal damping ratio with damper location. (a) Short cable; (b) long cable. · · · · Mode 1; – – – Mode 2; – · – Mode 3; - · · - Mode 4; —— Mode 5; in-plane $c_1 = c_2 = 0$, $k = 0$.

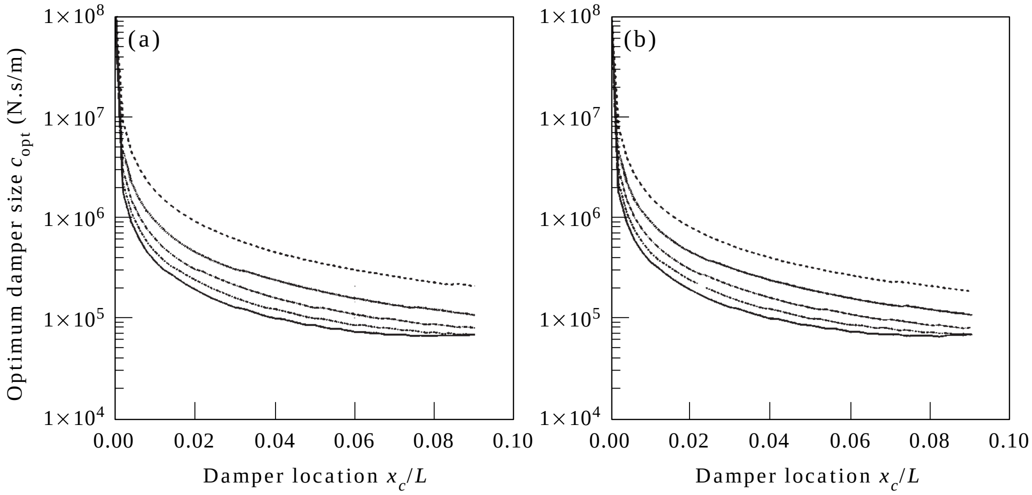

Figure 9. Variations of optimum damper size with damper location. (a) Short cable; (b) long cable. · · · · Mode 1; – – – Mode 2; – · – Mode 3; - · · - Mode 4; —— Mode 5, in-plane $c_1 = c_2 = 0$, $k = 0$.

slightly smaller than the short cable, as shown in Figure 9. The relationships between the maximum modal damping ratio and the damper position and between the optimum damper size and the damper position in the first five out-of-plane modes of the long cable are almost the same as those of the short cable.

4.2. EFFECTS OF DAMPER STIFFNESS

In practice, an oil damper may possess a certain amount of stiffness depending on the manufacture of the damper and the used oil. Effects of damper stiffness on the achievable maximum modal damping ratio, however, have not been investigated yet [4]. By using the hybrid method, the effects of damper stiffness can be easily examined. Figure 10(a) shows variations of the maximum modal damping ratios in the first five in-plane vibrational modes of the short cable with damper stiffness while Figure 10(b) displays the

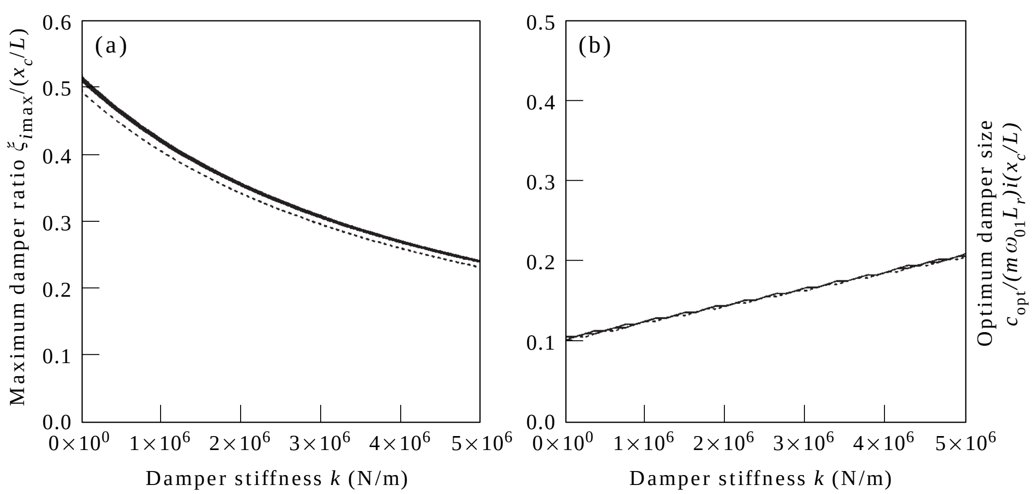

Figure 10. Effects of damper stiffness on maximum modal damping ratio and optimum damper size. (a) Damping ratio; (b) damper size; – – – Mode 1; —— other modes; short cable (in-plane) $x_c/L = 0{\cdot}02$, $c_1 = c_2 = 0$.

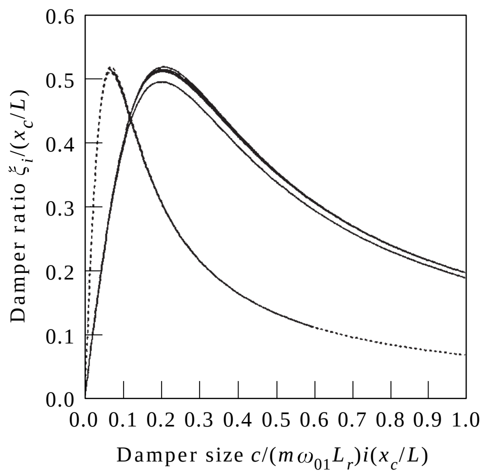

Figure 11. Effects of damper direction γ on modal damping ratio. —— In-plane modes; – – – out-of-plane modes; short cable, $x_c/L = 0{\cdot}02$, $\gamma = 30°$.

corresponding optimum damper size with damper stiffness. A pair of dampers considered here are symmetrically installed at a location x_c of $0{\cdot}02L$ from the deck support. Clearly, the existence of damper stiffness does affect the achievable maximum damping ratios in all concerned modes provided by the attached oil dampers. The greater damper stiffness causes the larger reduction of the achievable maximum modal damping ratio. These observations are also applied to the out-of-plane vibrational modes of the short cable as well as both in-plane and out-of-plane modes of the long cable. Thus, damper stiffness should be reduced as much as possible in the design and manufacture of oil dampers. If damper stiffness cannot be totally eliminated, the optimum damper size for the short cable should be selected according to Figure 10(b), which shows the greater damper stiffness needs the larger damper size in order to acheive the corresponding maximum modal damping ratio.

4.3. EFFECTS OF DAMPER DIRECTION γ

Section 2.2 reveals that when a pair of oil dampers are symmetrically installed to the short cable at a damper direction γ of 45°, the oil dampers of optimum damping coefficient can provide the same maximum modal damping ratios to both in-plane and out-of-plane vibrational modes. However, it is not clear how to select damper size if a pair of oil dampers are symmetrically installed but the damper direction γ is not 45°. Actually, the damper direction γ less than 45° is very common in practice.

Let us consider a case in which a pair of oil dampers are symmetrically installed to the short cable at a damper direction γ of 30°. Figure 11 shows the modal damping ratios in the first five in-plane and out-of-plane modes of vibration against the normalized damper size. It is seen that the modal damping ratio curves for the in-plane modes do not overlap those for the out-of-plane modes. The optimum damper size required for achieving the maximum out-of-plane modal damping ratio is only about one-third of that for the in-plane modes. Obviously, it is impossible for the oil dampers to achieve the maximum modal damping ratios in both in-plane and out-of-plane modes at the same time. Thus, a decision should be made by designers based on practical situation whether in-plane vibration or out-of-plane cable vibration should be mainly attenuated or both should be evenly mitigated.

To further understand the effects of damper direction γ, variations of the maximum modal damping ratio and the optimum damper size with the damper direction are computed and plotted in Figure 12 for the first five in-plane and out-of-plane modes of the short cable within a range of γ from 20° to 70°. It is seen that with the increase of damper direction γ, the optimum damper sizes for the in-plane modes become smaller while those for the out-of-plane modes become larger. At the damper direction γ of 45°, two curves cross each other, indicating that the dampers with the optimum damper size can provide the same maximum modal damping ratios for both in-plane and out-of-plane modes of vibration. Clearly, if the damper direction γ is less than 45° and mainly the attenuation of the in-plane vibration is required, the optimum damper size should be less than that at 45°. If mainly the attenuation of the out-of-plane vibration is needed, the larger damper size than that at 45° should be chosen. A similar analysis can be carried out for a damper direction γ larger than 45°.

The inclination of the short cable is 37·2° in the above discussion. Further studies show that the results obtained for the cable inclination of 37·2° are also valid for the cable inclination between 0° and 70°. For the long cable, due to cable sag effects variations of the optimum damper size with the damper direction γ are slightly different from the short cable. The optimum damper direction γ which can provide the maximum modal damping ratio at the same time for the first in-plane and out-of-plane modes of vibration is less than 45°, and in general the optimum damper direction γ becomes smaller as cable inclination reduces.

4.4. SENSITIVITY TO DAMPER DIRECTION α

To achieve the maximum modal damping ratio by using the smallest damper size, oil dampers should be installed normal to the cable. However, the exact normal direction α of the dampers to the cable may not be guaranteed in practice due to installation, but it is hoped that the small deviation from the normal damper direction α does not affect the achievement of the maximum modal damping ratios for the already-fixed damper size. A sensitivity study is thus carried out.

For the long cable, the cable inclination is 16° and the normal direction α of the oil dampers to the cable is 72°. Assume that the damper direction γ is 45° and a pair of oil

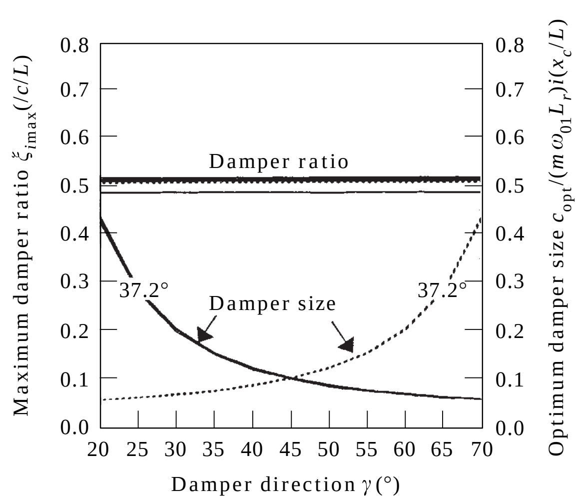

Figure 12. Effects of damper direction γ on maximum modal damping ratio and optimum damper size. —— In-plane; – – – out-of-plane; short cable, $x_c/L = 0{\cdot}02$.

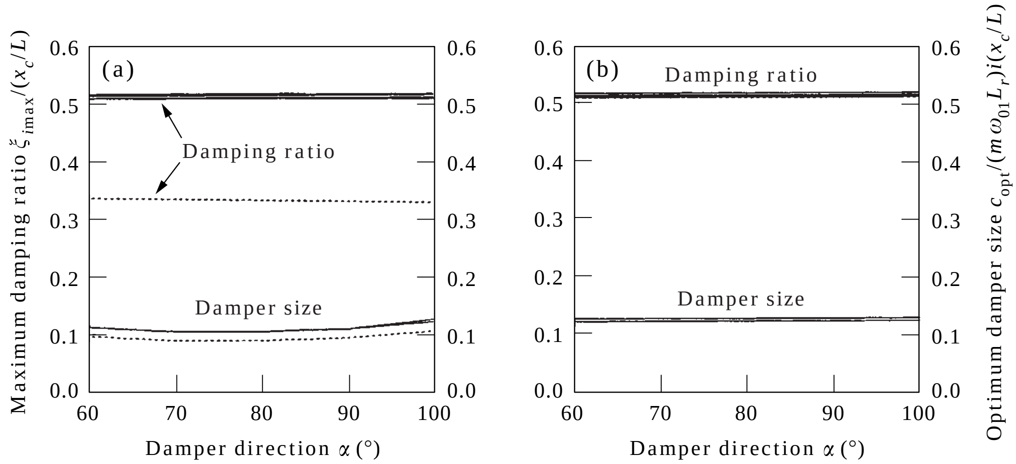

Figure 13. Sensitivities of maximum modal damping ratio and optimum damper size to damper direction α. (a) In-plane; (b) out-of-plane; – – – Mode 1; —— other modes; long cable, $x_c/L = 0{\cdot}02$.

dampers are symmetrically installed at a position of $x_c/L = 0{\cdot}02$, and then the damper direction α is changed from 60° to 100° to see variations of the maximum modal damping ratio and optimum damper size in both in-plane and out-of-plane modes of vibration. Figure 13(a) shows the maximum modal damping ratios and the optimum damper sizes of the in-plane vibration while Figure 13(b) displays the same quantities of the out-of-plane vibration. Clearly, within the studied range of damper direction α, the maximum damping ratios and the optimum damper sizes remain constant in the out-of-plane cable vibration and change only slightly in the in-plane cable vibration. It is also seen that the normal direction α of the oil dampers to the cable can be regarded as optimum direction in the sense that the maximum in-plane modal damping ratios can be achieved using the smallest damper size. From the above discussion, it can be concluded that the maximum modal damping ratio and the optimum damper size are not sensitive to the small deviation of damper direction α from the position normal to the cable.

4.5. SENSITIVITY TO DAMPER DIRECTION γ

For some cable stayed bridges, only single oil damper is installed to the cable in the cable plane to attenuate in-plane cable vibration only. The ideal damper direction γ is thus 90° according to the definition used in this study. Sensitivity studies are carried out to see the effects of small deviation of damper direction γ from 90° on the achievement of the maximum modal damping ratio and the optimum damper size. This kind of deviation may be caused by the improper installation or the out-of-plane wind force. Figure 14 shows the variations of the maximum modal damping ratio and optimum damper size in the first five in-plane vibrational modes of the short cable with the damper direction γ ranging from 80° to 100°. It is seen that the small deviation of the single damper from the cable plane does not affect the achievable maximum modal damping ratios as well as the corresponding optimum damper sizes obtained at the damper direction γ of 90°.

5. MULTI-PAIRS OF OIL DAMPERS

For the long cable studied here, due to sag effects the achievable maximum modal damping ratio in the first in-plane mode of vibration may not be high enough to overcome the cable vibration problem. This section thus aims to explore the possibility of enhancing

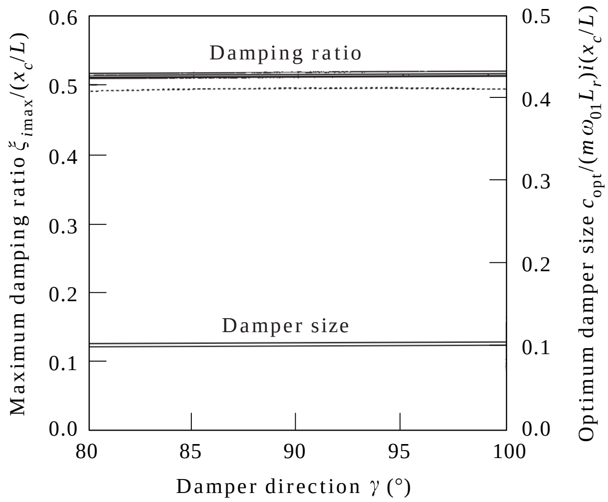

Figure 14. Sensitivities of maximum damping ratio and optimum damper size to damper direction γ. – – – Mode 1; —— other modes; short cable (in-plane), $x_c/L = 0{\cdot}02$.

the first in-plane maximum modal damping ratio using two pairs of oil dampers or a pair of oil dampers plus a single oil damper. The pair of oil dampers are considered to be installed symmetrically to the cable and the single damper is always installed in the cable plane. The damper direction γ for the pair of dampers is selected as 45° and the damper direction α for both the pair of oil dampers and the single oil damper is so selected that the dampers are always normal to the long cable.

5.1. TWO PAIRS OF DAMPERS AT SAME SIDE

Let us consider the case in which two pairs of oil dampers are installed to the long cable at the same side near the deck support at locations x_{c1} and x_{c2} of $0{\cdot}015L$ and $0{\cdot}020L$, respectively. Figures 15(a) and (b) show the modal damping ratio curves in the first five

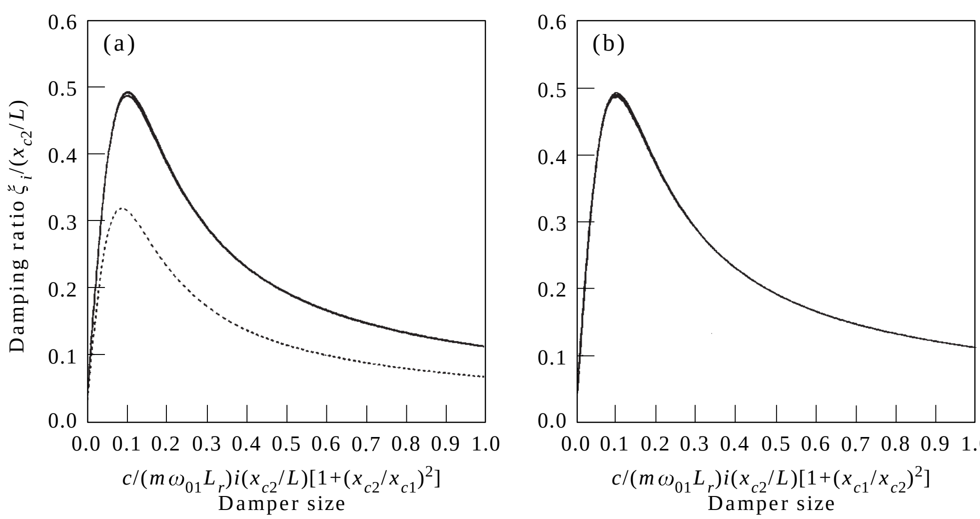

Figure 15. Modal damping ratio curve of long cable with two pairs of dampers at the same side. (a) In-plane; (b) out-of-plane; long cable, $x_{c1} = 0{\cdot}015L^{(L)}$, $x_{c2} = 0{\cdot}020L^{(L)}$.

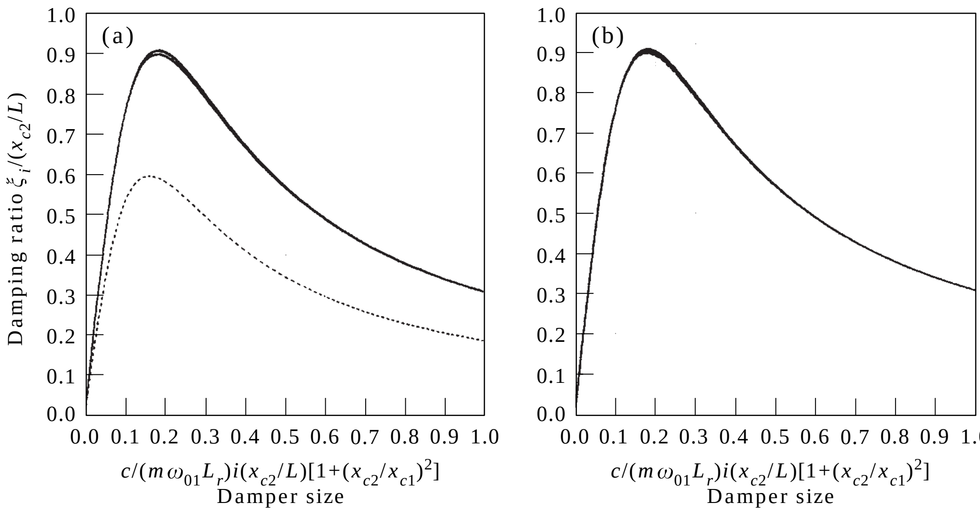

Figure 16. Modal damping ratio curve of long cable with two pairs of dampers at different sides. (a) In-plane; (b) out-of-plane; – – – Mode 1; —— other modes; long cable, $x_{c1} = 0{\cdot}015L^{(R)}$, $x_{c2} = 0{\cdot}020L^{(L)}$.

in-plane and out-of-plane vibration modes, respectively, of the long cable provided by the two pairs of oil dampers at the same side. Compared with Figures 4(a) and (b), one can find that both in-plane and out-of-plane modal damping ratios provided by the two pairs of dampers are almost the same as those by one pair of oil dampers. This means that the installation of two pairs of oil dampers at the same side near the deck support cannot enhance the in-plane and out-of-plane modal damping ratios and thus cannot overcome the frequency avoidance problem occurring in the first in-plane vibrational mode of the long cable. Similar results are also obtained for the short cable. Note that the normalization of damper size in Figure 15 is different from that in Figure 4. Thus, the optimum damper size required to achieve the maximum modal damping ratio from the two pairs of oil dampers is smaller than that from one pair of oil dampers.

5.2. TWO PAIRS OF DAMPERS AT DIFFERENT SIDES

Let us retain one pair of oil dampers at the location x_{c2} of $0{\cdot}020L$ near the deck support and move another pair of oil dampers to a location x_{c1} of $0{\cdot}015L$ near the tower support. The modal damping ratios in the first five in-plane and out-of-plane modes are calculated for this case. It is interesting to find that both in-plane and out-of-plane modal damping ratios are significantly increased, as shown in Figures 16(a) and (b). In particular, the achievable maximum normalized modal damping ratios in the first five out-of-plane and the four higher in-plane vibrational modes are increased to 0·9 compared with 0·5 from one pair of oil dampers. Though the installation of two pairs of oil dampers at the two sides of the long cable cannot overcome the frequency avoidance problem in the first in-plane vibrational mode, the achievable normalized maximum modal damping ratio in that modal is, however, increased to 0·6. This maximum modal damping ratio is equivalent to 1·2% of critical damping ratio and it may be large enough to overcome wind or wind-rain-induced cable vibration [10]. Further studies show that the maximum modal damping ratio provided by the two pairs of oil dampers located at different sides of the cable is approximately equal to the sum of the maximum modal damping ratios provided each pair of oil dampers. The optimum damper size required for achieving the maximum modal damping ratio in the case of two pairs of dampers located at different sides is almost the same as that in the case of one pair of dampers, which can be explained in terms of the different normalization used in Figures 4 and 16.

Forced vibration analysis is now performed to see the effectiveness of two pairs of oil dampers located at different sides through the comparison with two pairs of dampers installed at the same side. The damper sizes used for both cases are of their own optimum values. The uniformly distributed harmonic load of intensity 1 N/m is applied normal to the cable in both the x–y plane and x_1–z plane (see Figure 1). Figures 17(a) and (b) show the in-plane and out-of-plane maximum displacement response amplitudes, respectively, of the long cable with two pairs of oil dampers located at the same side and at different sides. In the two pairs of oil dampers, one pair of dampers is located at $0{\cdot}015L$ from the cable support and the other pair is at $0{\cdot}02L$ from the cable support. It is seen that three resonant peaks corresponding to the first three symmetric modes of vibration are significantly reduced in not only the out-of-plane vibration but also the in-plane vibration when the two pairs of dampers are installed at different sides. The significant reduction of the resonant peak in the first in-plane vibrational mode indicates that no matter whether the modal damping ratio is affected by frequency avoidance or not, the installation of two pairs of dampers at different sides can almost double the maximum modal damping ratio provided by a single pair of dampers. Thus, it is possible to remedy insufficient in-plane modal damping ratio due to frequency avoidance by installing two pairs of dampers at different ends of the cable.

5.3. A PAIR OF DAMPERS PLUS A SINGLE DAMPER

For actual cables in cable stayed bridges, it may be difficult to install a pair of dampers herringbonely between the cable and bridge tower. However, it is possible to install a single oil damper between the cable and bridge tower in the cable plane while a pair of oil dampers are still herringbonely installed between the cable and bridge deck. In this case, the in-plane resultant damping forces provided by all three dampers are different from the out-of-plane resultant damping forces provided by only two dampers near the bridge deck. The maximum in-plane and out-of-plane modal damping ratios thus vary with the damper direction γ of the pair of dampers. Let the single damper be located at x_{c1} of $0{\cdot}015L$ from

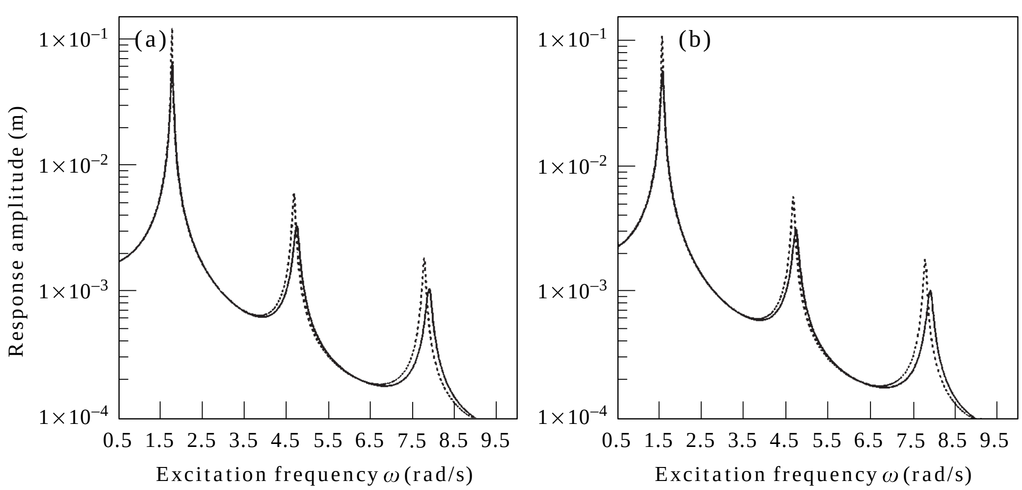

Figure 17. Maximum dynamic response of long cable with two pairs of dampers at different sides and same side. (a) In-plane; (b) out-of-plane; – – – At same side; —— at different sides; long cable, $c^1 = 4{\cdot}92 \times 10^5$, $c^2 = 9{\cdot}38 \times 10^5$, $x_{c1}^1 = 0{\cdot}015^{(L)}$, $x_{c2}^1 = 0{\cdot}02^{(L)}$, $x_{c1}^2 = 0{\cdot}015^{(R)}$, $x_{c2}^2 = 0{\cdot}02^{(L)}$.

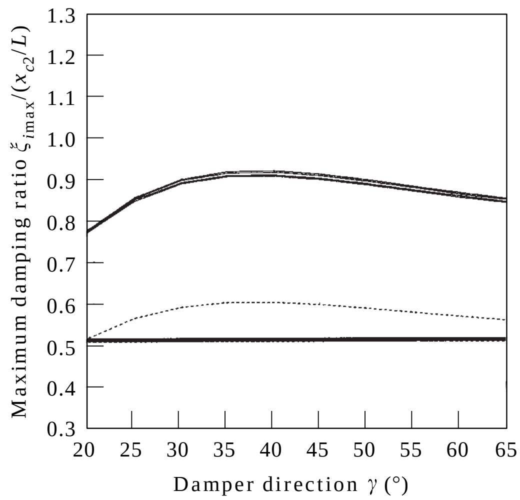

Figure 18. Maximum damping ratios of long cable with one pair of dampers plus single damper at different sides. – – – Mode 1; —— other modes; long cable, $x_{c1} = 0{\cdot}015L^{(R)}$, $x_{c2} = 0{\cdot}02^{(L)}$.

the tower support of the cable and the pair of oil dampers symmetrically located at x_{c2} of $0{\cdot}02L$ from the bridge support of the cable. Then change the damper direction γ of the pair of dampers to see how the maximum modal damping ratios alter. The results are shown in Figure 18, from which it is seen that the in-plane maximum modal damping ratio does vary with damper direction γ whereas the out-of-plane maximum modal damping ratio remains constant. The optimum damper direction γ ranges from 35° to 45°. It is also seen that even though the achievable normalized maximum modal damping ratio in the first in-plane mode is much smaller than those in the higher in-plane modes, it is still greater than 0·5 in the out-of-plane modes of vibration when the damper direction γ is within 35° to 45°. This indicates that the arrangement of three dampers in the way described here can also overcome the problem caused by frequency avoidance.

6. CONCLUSIONS

The hybrid method developed in part I of this paper for three dimensional free and forced vibrations of an inclined sag cable with multi-pairs of oil dampers has been applied to two typical cables in a real cable stayed bridge. For a taut stay cable, the installation of oil dampers can significantly increase both in-plane and out-of-plane modal damping ratios and thus reduce dynamic responses if damper properties and directions are selected properly. For a long cable with significant sag, even though damper properties and directions are properly selected, the achievable maximum modal damping ratio in the first in-plane vibrational mode is much smaller than those in the higher in-plane and all out-of-plane modes of vibration due to cable frequency avoidance effects. The cable inclination does not affect the maximum modal damping ratios and the optimum damper sizes for both in-plane and out-of-plane vibrational modes except for the first in-plane vibrational mode of the long cable with significant sag, in which the frequency avoidance effect is gradually attenuated as cable inclination increases.

Damper location, damper stiffness and damper direction also affect damper performance. The achievable maximum modal ratios increase as the damper location moves from the cable support but this has a limitation in practice. The damper stiffness always reduces damper performance so that it should be eliminated. All dampers should be installed normal to the cable but the damper performance will not be affected by small deviations of damper direction α from the normal position. The selection of damper direction γ depends on real situation. If the same in-plane and out-of-plane maximum modal damping ratios are required to be achieved at the same time, the damper direction γ should be 45°. To enhance insufficient modal damping ratio in the first in-plane mode of the long cable due to frequency avoidance effects, two pairs of oil dampers can be installed to the cable at different ends.

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